What will we do?

Why do we have programming languages?

Is programming language a computer language?

What compilers do you know?

Compilers, interpreters.

Keep a translator nearby to translate a paper each time.

Why compiling, why interpreting?

Pros and cons

https://jaxenter.com/energy-efficie nt-programming-languages-13726 4.html

Total

	Energy		Time		Mb
(c) C	1.00	(c) C	1.00	(c) Pascal	1.00
(c) Rust	1.03	(c) Rust	1.04	(c) Go	1.05
(c) C++	1.34	(c) C++	1.56	(c) C	1.17
(c) Ada	1.70	(c) Ada	1.85	(c) Fortran	1.24
(v) Java	1.98	(v) Java	1.89	(c) C++	1.34
(c) Pascal	2.14	(c) Chapel	2.14	(c) Ada	1.47
(c) Chapel	2.18	(c) Go	2.83	(c) Rust	1.54
(v) Lisp	2.27	(c) Pascal	3.02	(v) Lisp	1.92
(c) Ocaml	2.40	(c) Ocaml	3.09	(c) Haskell	2.45
(c) Fortran	2.52	(v) C#	3.14	(i) PHP	2.57
(c) Swift	2.79	(v) Lisp	3.40	(c) Swift	2.71
(c) Haskell	3.10	(c) Haskell	3.55	(i) Python	2.80
(v) C#	3.14	(c) Swift	4.20	(c) Ocaml	2.82
(c) Go	3.23	(c) Fortran	4.20	(v) C#	2.85
(i) Dart	3.83	(v) F#	6.30	(i) Hack	3.34
(v) F#	4.13	(i) JavaScript	6.52	(v) Racket	3.52
(i) JavaScript	4.45	(i) Dart	6.67	(i) Ruby	3.97
(v) Racket	7.91	(v) Racket	11.27	(c) Chapel	4.00
(i) TypeScript	21.50	(i) Hack	26.99	(v) F#	4.25
(i) Hack	24.02	(i) PHP	27.64	(i) JavaScript	4.59
(i) PHP	29.30	(v) Erlang	36.71	(i) TypeScript	4.69
(v) Erlang	42.23	(i) Jruby	43.44	(v) Java	6.01
(i) Lua	45.98	(i) TypeScript	46.20	(i) Perl	6.62
(i) Jruby	46.54	(i) Ruby	59.34	(i) Lua	6.72
(i) Ruby	69.91	(i) Perl	65.79	(v) Erlang	7.20
(i) Python	75.88	(i) Python	71.90	(i) Dart	8.64
(i) Perl	79.58	(i) Lua	82.91	(i) Jruby	19.84

Time & Memory	Energy & Time	Energy & Memory	Energy & Time & Memory
C • Pascal • Go	С	C • Pascal	C • Pascal • Go
Rust • C++ • Fortran	Rust	Rust • C++ • Fortran • Go	Rust • C++ • Fortran
Ada	C++	Ada	Ada
Java • Chapel • Lisp • Ocaml	Ada	Java • Chapel • Lisp	Java • Chapel • Lisp • Ocaml
Haskell • C#	Java	OCaml • Swift • Haskell	Swift • Haskell • C#
Swift • PHP	Pascal · Chapel	C# • PHP	Dart • F# • Racket • Hack • PHP
F# • Racket • Hack • Python	Lisp • Ocaml • Go	Dart • F# • Racket • Hack • Python	JavaScript • Ruby • Python
JavaScript • Ruby	Fortran • Haskell • C#	JavaScript • Ruby	TypeScript • Erlang
Dart • TypeScript • Erlang	Swift	TypeScript	Lua • JRuby • Perl
JRuby • Perl	Dart • F#	Erlang • Lua • Perl	
Lua	JavaScript	JRuby	
	Racket		
	TypeScript • Hack		
	PHP		
	Erlang		
	Lua • JRuby		
	Ruby		

Table 5. Pareto optimal sets for different combination of objectives.

What is assembler?



What do we deal with in asm? What do we deal with in PL?

Asm:registers

Asm:

- Registers
- Addresses

Asm:

- Registers
- Addresses
- CPU instructions (mov, add)

Asm:

- Registers
- Addresses
- CPU instructions (mov, add)

HL Programming languagesFunctions

- Functions
- Methods

- Functions
- Methods
- Variables

- Functions
- Methods
- Variables
- Data structures

- Functions
- Methods
- Variables
- Data structures
- loops

- Functions
- Methods
- Variables
- Data structures
- Loops
- modules

Why don't we write in asm? (or what do we write in asm?)

Std question:

Why learn other language if we can do everything in c++?

Are there languages, that cannot be compiled?

Who knows eval() function?

Cross compilers

JIT

Binary compilers (binary to binary translation)

X86 -> MIPS 6502 -> x86 68000 -> SPARC

Bratman's T diagrams

Src lang Target lang

Instrumental lang






How first compilers were created?

What is bootstrapping?

Compilation process







Metalanguages

Our spoken language is metalanguage.

Our spoken language is metalanguage.

Language and syntax

sentence = subject predicate.

sentence = subject predicate.

subject = "John" | "Mary".
predicate = "eats" | "talks".

sentence = subject predicate.

John eatsMary eatsJohn talksMary talks

S = AB.

$$S = AB.$$

 $L = \{ac, ad, bc, bd\}$

$$S = AB.$$

Language contains four sentences. Typically, language contains infinitely many sentences. *Infinite set may be defined with finite number of equations. ^{<i>ϕ*} - empty sequence.

S = A.

Infinite set may be defined with finite number of equations. ∅ - empty sequence.

S = A.

A = "a" A | ∅

Infinite set may be defined with finite number of equations. ^{<i>ϕ} - empty sequence.

S = A.

Nested sequences.

S = A.

Nested sequences.

S = A.

A = "a" A "c" | "b".

Nested sequences.

S = A.

$$L = \{b, abc, aabcc, aaabccc, ... \}$$

Structure of expressions E: expression; T: term; F: factor: V: variable;

Syntax tree



• Set of terminal symbols.

- Set of terminal symbols.
- Set of nonterminal symbols.

- Set of terminal symbols.
- Set of nonterminal symbols.
- Set of syntactic equations (productions)

- Set of terminal symbols.
- Set of nonterminal symbols.
- Set of syntactic equations (productions)
- Start symbol

Backus Naur form, BNF, 1960.

```
Syntax = production syntax | \varnothing.
production = identifier "=" expression ".".
expression = term | expression "|" term.
Term = factor | term factor.
Factor = identifier | string.
Identifier = letter | identifier letter | identifier
digit.
string = stringhead """.
stringhead = """ | stringhead character.
Letter = "A" | ... | "Z".
Digit = "0" | ... | "9".
```

Extended Backus Naur form, EBNF, 1977. By N. Wirth.

```
syntax = {production}.
production = identifier "=" expression ".".
expression = term {"|" term}.
term = factor {factor}.
Factor = identifier | string | "(" expression ")
" | "[" expression "]" | "{" expression "}".
Identifier = letter {letter | digit}.
String = """ {character} """.
Letter = "A" | ... | "Z".
Digit = "0" | ... | "9".
```

{x} is equivalent to an arbitrarily long sequence of x, therefore

{x} is equivalent to an arbitrarily long sequence of x, therefore

$A = A B \mid \varnothing$

{x} is equivalent to an arbitrarily long sequence of x, therefore

$A = A B \mid \varnothing$

Can be formulated as

A factor of the form [x] is equivalent to "x or nothing".
A factor of the form [x] is equivalent to "x or nothing".

Need for the special symbol $\ensuremath{\varnothing}$ vanishes.

US postal address in BNF

```
<postal-address> ::= <name-part> <street-address> <zip-part>
```

```
<name-part> ::= <personal-part> <last-name> <opt-suffix-part> <EOL>
| <personal-part> <name-part>
```

```
<personal-part> ::= <initial> "." | <first-name>
```

```
<street-address> ::= <house-num> <street-name> <opt-apt-num> <EOL>
```

```
<zip-part> ::= <town-name> "," <state-code> <ZIP-code> <EOL>
```

```
<opt-suffix-part> ::= "Sr." | "Jr." | <roman-numeral> | ""
<opt-apt-num> ::= <apt-num> | ""
```

digit excluding zero = "1" | "2" | "3" | "4" | "5" | "6" | "7" | "8" | "9" ; digit = "0" | digit excluding zero ; Context free languages are languages that are describable by a context free grammar. So the question reduces to : *What is a context free grammar?* A language is a set of strings, where each string is built up from a base set of allowable symbols, the alphabet (Σ). Each such string is called a sentence of the language. This includes the empty sentence of no symbols at all. For most languages, not just any sequences of symbols form a sentence, but there are certain structural rules that define the allowable sentences. These structural rules, taken together, form what is known as the grammar of the language. For most languages, not just any sequences of symbols form a sentence, but there are certain structural rules that define the allowable sentences. These structural rules, taken together, form what is known as the grammar of the language.

It is important here to realize that, while a specific grammar, uniquely determines a language, the converse is not true in general. A given language, might have many different grammars that describe it. For most languages, not just any sequences of symbols form a sentence, but there are certain structural rules that define the allowable sentences. These structural rules, taken together, form what is known as the grammar of the language.

It is important here to realize that, while a specific grammar, uniquely determines a language, the converse is not true in general. A given language, might have many different grammars that describe it. Note also, that nowhere in the above have I mentioned any concept of meaning for the sentences of the language. Formal language theory does not address meaning, only structure.

Note also, that nowhere in the above have I mentioned any concept of meaning for the sentences of the language. Formal language theory does not address meaning, only structure.

For example, on this view, "*The invisible, pink unicorn ate wrathful balls, after sleeping diligently*" is a valid English sentence, although perhaps meaningless, because it respects the grammar of the English language.

Note also, that nowhere in the above have I mentioned any concept of meaning for the sentences of the language. Formal language theory does not address meaning, only structure.

For example, on this view, "*The invisible, pink unicorn ate wrathful balls, after sleeping diligently*" is a valid English sentence, although perhaps meaningless, because it respects the grammar of the English language.

```
<statement> = <subject> <action verb> <indirect object>
<object> | <subject> <linking verb> <adjective>
<subject> = Ann | Bob
<verb> = throws | gives | takes
<indirect object> = Ann | Bob | him | her
<object> = the ball | the sock
<linking verb> = is
<adjective> = female | male | tired | sleepy
```

```
<statement> = <subject> <action verb> <indirect object>
<object> | <subject> <linking verb> <adjective>
<subject> = Ann | Bob
<verb> = throws | gives | takes
<indirect object> = Ann | Bob | him | her
<object> = the ball | the sock
<linking verb> = is
<adjective> = female | male | tired | sleepy
```

This looks like a possible road to representing language, but it has a problem, which is expressed by the term "context-free". That is, you can't build a finite context-free language which can hold certain types of relations between things. We can't build a language where, say, saying "Ann is female." would imply that in order to speak of Ann as an indirect object later on, we have to use "her".

a set of production rules that describe all possible strings in a given formal language. Production rules are simple replacements. Recoursive descent parser

• Before the parsing starts, first symbol of the entity we parse is the current

Recoursive descent parser

- Before the parsing starts, first symbol of the entity we parse is the current
- Function reads all symbols of the current non-terminal, or emits error

- Before the parsing starts, first symbol of the entity we parse is the current
- Function reads all symbols of the current non-terminal, or emits error
- If rules for this non-terminal contain other non-terminals in right side of equation, then function calls functions to call those non-terminals

- Before the parsing starts, first symbol of the entity we parse is the current
- Function reads all symbols of the current non-terminal, or emits error
- If rules for this non-terminal contain other non-terminals in right side of equation, then function calls functions to call those non-terminals
- Then first symbol of the next construct is current.



Current symbol at the start and the end of the parsing function of non-terminal A.

therefore

NextCh(); - read the first symbol in to variable ch.

therefore

NextCh(); - read the first symbol
in to variable ch.

S(); - function of parsing a
beginning of non-terminal;

therefore

NextCh(); - read the first symbol in to variable ch.

S(); - function of parsing a beginning
of non-terminal;

If ch # eot then begin error() end;

Regular languages

Regular languages

Syntactic equations of the form defined in EBNF generate context-free languages.

Regular languages

Syntactic equations of the form defined in EBNF generate context-free languages.

Substitution of the symbol left of = by a sequence derived from the expression to the right of = is always permitted, regardless of the context in which the symbol is embedded within the sentence.

- Machine language
- Assembly language
- High level language

10	ORG	\$4000			
11 A1		\$3C			
12 A2	-	\$3E			
13 A4		\$42			
14 AUXMOVE		\$C311			
15					
16	•••••	•••••	•••		
17 · SETUP	- 000	e data for V	TOC		
18 · and c	atalog	to auxmem a			
19 . 6000-	BJFF (pseudo trk 1			
20 • 0-3)					
21	••••••	••••••••	•		
22 SETUP	LUA	# <vioc< td=""><td></td><td></td><td></td></vioc<>			
23	SUA	A1			
		SVICE			
	214				
		A		1.1	
		AL END			
20	17	1241		ALC: NOT	A DECT
*				who seems to be	
THE REPORT				A COMPANY	Contraction of the local division of the loc
		Anna and			
		THE REAL PROPERTY	4 19		
		Elander and		COMPANY OF THE OWNER	

https://www.masswerk.at/6502/6502_instruction_set.html

Machine Language vs. Assembly Language

Objective: Multiply the value stored in R4 by 12

Assembly Language .ORIG x3000 AND R0, R0, #0 AND R7, R7, #0 ADD R0, R0, #12 TEST BRnz DONE ADD R7, R7, R4 ADD R0, R0, #-1 BRnzp TEST DONE HALT 7-3 .END

Von Neumann architecture

Von Neumann architecture





Memory Layout (Virtual address space of a C process)







Von Neumann architecture

• In case of von Neumann architecture, instructions and data are stored in the same memory, so instructions are fetched over the same data path used to fetch data.
- In case of von Neumann architecture, instructions and data are stored in the same memory, so instructions are fetched over the same data path used to fetch data.
- This means that a CPU cannot simultaneously read an instruction and read or write data from or to the memory.

- In case of von Neumann architecture, instructions and data are stored in the same memory, so instructions are fetched over the same data path used to fetch data.
- This means that a CPU cannot simultaneously read an instruction and read or write data from or to the memory.

Harvard architecture

 In a computer using the Harvard architecture, the CPU can both read an instruction and perform a data memory access at the same time, even without a cache. Harvard architecture

- In a computer using the Harvard architecture, the CPU can both read an instruction and perform a data memory access at the same time, even without a cache.
- A Harvard architecture computer can thus be faster for a given circuit complexity because instruction fetches and data access do not contend for a single memory pathway.

Harvard architecture

- In a computer using the Harvard architecture, the CPU can both read an instruction and perform a data memory access at the same time, even without a cache.
- A Harvard architecture computer can thus be faster for a given circuit complexity because instruction fetches and data access do not contend for a single memory pathway.
- has distinct code and data address spaces: instruction address zero is not the same as data address zero. Instruction address zero might identify a twenty-four bit value, while data address zero might indicate an eight-bit byte







Two main parts

• CPU

Two main parts

- CPU
- Memory

Two main parts

- CPU
- Memory





			1		10					5			1 1 1 1			
																F
	+	-	-	-	-	-	-	-	-	-		-	-	-	-	-
	-	-	_		-	-	-	-	-	_	-	-	-	-	-	-
			- 3	1							2					
											1	1				
	-	-				-	-	-								
		-	_		-	-	-	-	-	-	-	-	-	-	-	-
									1							
	-						-				1					1
	-	-	-		-	-	-	-		-	-	-	-	-	-	-
$ \rightarrow $	_		_	_	_	-	-	-		_	_	-	-	-	-	-
											-	1				
				1							1	1				
										-	1	1				
	+	-			-	-	-	-					-	-		-
	-	-	_	-	-	-	-	-	-	-		-	-	-	-	-
	_	-	_					_				_	L	_		
				10												
					-											
	-	-			-	-	-	-		-		-	-	-	-	-
	-	-	_	-	_	-	-	-	-	_	-	_	-	-	-	-
					_											
												1				
	-	1		-	-	-	-	-			-		-	-		-
	-	-	_		-	-	-	-	-	-	-	-	-	-	-	-
	_				_	-				_		_	_			-



Instruction Set Architecture. An agreement between hardware and human for making interaction. Example : ADD R1, R2, R3 Can be represented as :

ISA Classification

Two major schools of ISA



Complex Instruction Set Computer



Reduced Instruction Set Computer

CISC Philosophy

Reduce amount of storage used and accessed - reduce load/store.

Give support for compatibility.

Make compiler's job easier.

Support complex assembly level programming.

RISC Philosophy

- Execute one instruction per clock.
- Keep all instructions of same size.

 Allow only load / store instruction to access the memory.
Give support for high level languages (like C, C++, Java).









• Program counter

- Program counter
- Instruction decoder

- Program counter
- Instruction decoder
- Data bus

- Program counter
- Instruction decoder
- Data bus
- General purpose registers

- Program counter
- Instruction decoder
- Data bus
- General purpose registers
- Arithmetic and logic unit

A pictorial representation of some of the registers in a 32-bit processor. Note that each register is composed of 32 chambers containing either zero or one. Since there are only two possible values a chamber can have and since there are 32 chambers in a register, a single register can have (2 raised to the power 32) different combinations of 0s and 1s (i.e. =2^32 different values).

EAX



Endianness 0x12345678

Big Endian

Little Endian

Address	Value		
X+0	12		
X+1	34		
X+2	56		
X+3	78		

Address	Value
X+0	78
X+1	56
X+2	34
X+3	12



Little Endian

MIPS vs. Intel

bitcount: ADD r2, r0, r0 ADD r3, r0, r0 loop: ADD1 r4, r0, #1 SHLL r4, r4, r2 AND r4, r4, r1 **BEQZ** noadd ADDI r2, r0, #1 noadd: ADDI r2, r0, #1 SUBI r0, r2, #32 **BNEZ** loop RET

bitcount: **# EAX->EBX** XOR EBX, EBX XOR CL. CL loop: MOVEDX, #1 SHL EDX, CL AND EDX. EAX BEQ noadd INC EBX noadd: INC CL CMP CL, #32 BNE loop RET

CISC (it is a retronym)

• Complex instruction-decoding logic, driven by the need for a single instruction to support multiple addressing modes.

- Complex instruction-decoding logic, driven by the need for a single instruction to support multiple addressing modes.
- Small number of general purpose registers. Instructions which operate directly on memory, and only the limited amount of chip space is dedicated for general purpose registers.

- Complex instruction-decoding logic, driven by the need for a single instruction to support multiple addressing modes.
- Small number of general purpose registers. Instructions which operate directly on memory, and only the limited amount of chip space is dedicated for general purpose registers.
- Several special purpose registers. Many CISC designs set aside special registers for the stack pointer, interrupt handling, and so on. This can simplify the hardware design somewhat, at the expense of making the instruction set more complex.

- Complex instruction-decoding logic, driven by the need for a single instruction to support multiple addressing modes.
- Small number of general purpose registers. Instructions which operate directly on memory, and only the limited amount of chip space is dedicated for general purpose registers.
- Several special purpose registers. Many CISC designs set aside special registers for the stack pointer, interrupt handling, and so on. This can simplify the hardware design somewhat, at the expense of making the instruction set more complex.
- 'Condition code" register. This register reflects whether the result of the last operation is less than, equal to, or greater than zero and records if certain error conditions occur.
RISC

• One Cycle Execution Time: RISC processors have a CPI (clock per instruction) of one cycle.

RISC

- One Cycle Execution Time: RISC processors have a CPI (clock per instruction) of one cycle.
- Pipelining: A technique that allows simultaneous execution of parts, or stages, of instructions to more efficiently process instructions.

RISC

- One Cycle Execution Time: RISC processors have a CPI (clock per instruction) of one cycle.
- Pipelining: A technique that allows simultaneous execution of parts, or stages, of instructions to more efficiently process instructions.
- Large Number of Registers. The RISC design philosophy generally incorporates a larger number of registers to prevent large amounts of interactions with memory.

Architectural Characterstics	Complex Instruction Set Computer(CISC)	Reduced Instruction Set Computer(RISC)
Instruction size and	Large set of instructions with variable formats	Small set of instructions with
format	(16-64 bits per instruction).	fixed format (32 bit).
Data transfer	Memory to memory.	Register to register.
CPU control	Most micro coded using control memory (ROM)	Mostly hardwired without
	but modern CISC use hardwired control.	control memory.
Instruction type	Not register based instructions.	Register based instructions.
Memory access	More memory access.	Less memory access.
Clocks	Includes multi-clocks.	Includes single clock.
Instruction nature	Instructions are complex.	Instructions are reduced and simple.

CISC	RISC	
The original microprocessor ISA	Redesigned ISA that emerged in the early 1980s	
Instructions can take several clock cycles	Single-cycle instructions	
Hardware-centric design	Software-centric design	
 the ISA does as much as possible using hardware circuitry 	 High-level compilers take on most of the burden of coding many software steps from the programmer 	
More efficient use of RAM than RISC	Heavy use of RAM (can cause bottlenecks if RAM is limited)	
Complex and variable length instructions	Simple, standardized instructions	
May support microcode (micro- programming where instructions are treated like small programs)	Only one layer of instructions	
Large number of instructions	Small number of fixed-length instructions	
Compound addressing modes	Limited addressing modes	

RISC vs. CISC

Parameter	RISC	CISC
Instruction types	Simple	Complex
 Number of instructions 	Reduced (30-40)	Extended (100-200)
Duration of an instruction	One cycle	More cycles (4-120)
Instruction format	Fixed	Variable
Instruction execution	In parallel (pipeline)	Sequential
Addressing modes	Simple	Complex
Instructions accessing the memory	Two: Load and Store	Almost all from the set
Register set	multiple	unique
Complexity	In compiler	In CPU (micro-program)



https://en.wikipedia.org/wiki/Addressing_mode

https://en.wikipedia.org/wiki/Addressing_mode

Direct addressing mode:

movl ADDRESS, %eax

This loads %eax with the value at memory address ADDRESS .

https://en.wikipedia.org/wiki/Addressing_mode

Direct addressing mode:

movl ADDRESS, %eax

This loads %eax with the value at memory address ADDRESS.

https://en.wikipedia.org/wiki/Addressing_mode

Direct addressing mode:

movl ADDRESS, %eax

This loads %eax with the value at memory address ADDRESS.

Indirect addressing mode: movl (%eax), %ebx

Eax helds an address, and we move the value at that address to ebx.

https://en.wikipedia.org/wiki/Addressing_mode

Direct addressing mode:

movl ADDRESS, %eax

This loads %eax with the value at memory address ADDRESS.

Indirect addressing mode: movl (%eax), %ebx

%eax helds an address, and we move the value at that address to %ebx.

Base pointer addressing mode:

movl 4(%eax), %ebx

As indirect, but adds a constant value to the address in the register. Useful for record fields.

https://en.wikipedia.org/wiki/Addressing_mode

Immediate addressing mode:

movl \$12, %eax If we did not use \$ sign, then the value located at memory location 12 would be used.

https://en.wikipedia.org/wiki/Addressing_mode

Immediate addressing mode:

movl \$12, %eax If we did not use \$ sign, then the value located at memory location 12 would be used.

Register addressing mode:

simply moves data in or out of a register. In all of our examples, register addressing mode was used for the other operand.

https://en.wikipedia.org/wiki/Addressing_mode

Immediate addressing mode:

movl \$12, %eax If we did not use \$ sign, then the value located at memory location 12 would be used.

Register addressing mode:

simply moves data in or out of a register. In all of our examples, register addressing mode was used for the other operand.

Every mode except immediate mode can be used as either the source or destination operand. Immediate mode can only be a source operand.



Layout of the %eax register

function name - represents the address where the function's code starts. (in asm - label)

function name - represents the address where the function's code starts. (in asm - label)

function parameters.

function name - represents the address where the function's code starts. (in asm - label)

function parameters.

local variables

function name - represents the address where the function's code starts. (in asm - label)

function parameters.

local variables

static variables

function name - represents the address where the function's code starts. (in asm - label)

function parameters.

local variables

static variables

global variables

function name - represents the address where the function's code starts. (in asm - label)

function parameters.

local variables

static variables

global variables

return address

function name - represents the address where the function's code starts. (in asm - label)

function parameters.

local variables

static variables

global variables

return address

return value

Calling conventions.

```
byte byte1 = 150; // 10010110
byte byte2 = 199; // 11000111
```

```
byte byte1 = 150; // 10010110
byte byte2 = 199; // 11000111
```

byte byte3 = byte1 + byte2;

```
byte byte1 = 150; // 10010110
byte byte2 = 199; // 11000111
```

byte byte3 = byte1 + byte2;

byte3 = 94

```
byte byte1 = 150; // 10010110
byte byte2 = 199; // 11000111
```

byte
$$3 = 94$$

```
150+199=349, binary 1 0101 1101,
the upper 1 bit is dropped and the byte becomes
0101 1101;
```

Clock arithmetic

- **3:00** + 2 hours = **5:00**
- **3:00** + 14 hours = **5:00**
- **3:00** 10 hours = **5:00**

Clock arithmetic

- **3:00** + 2 hours = **5:00**
- 3:00 + 14 hours = 5:00
- 3:00 10 hours = 5:00

In the "clock arithmetic", 2, 14 and –10 are just three different ways to write down the same number.

Clock arithmetic

- **3:00** + 2 hours = **5:00**
- 3:00 + 14 hours = 5:00

In the "clock arithmetic", 2, 14 and –10 are just three different ways to write down the same number.

3:00 - 10 hours = **5:00**

They are interchangeable in multiplications too:

```
12:00 + (3 * 2) hours = 5:00
12:00 + (3 * 14) hours = 5:00
12:00 + (3 * -10) hours = 5:00
```

Clock arithmetic

- **3:00** + 2 hours = **5:00**
- 3:00 + 14 hours = 5:00

In the "clock arithmetic", 2, 14 and –10 are just three different ways to write down the same number.

3:00 - 10 hours = **5:00**

They are interchangeable in multiplications too:

12:00 + (3 * 2) hours = 5:00
12:00 + (3 * 14) hours = 5:00
12:00 + (3 * -10) hours = 5:00

formal term for "clock arithmetic" is "modular arithmetic". In modular arithmetic, two numbers are equivalent if they leave the same non-negative remainder when divided by a particular number.

consider 3-bit integers, which can represent integers from 0 to 7.

If you add or multiply two of these 3-bit numbers in fixed-width binary arithmetic, you'll get the "modular arithmetic" answer:

- 1 + 2 -> 3
- 4 + 5 -> 1

consider 3-bit integers, which can represent integers from 0 to 7.

If you add or multiply two of these 3-bit numbers in fixed-width binary arithmetic, you'll get the "modular arithmetic" answer:

- 1 + 2 -> 3
- 4 + 5 -> 1

The calculations wrap around, because any answer larger than 7 cannot be represented with 3 bits. The wrapped-around answer is still meaningful:
consider 3-bit integers, which can represent integers from 0 to 7.

If you add or multiply two of these 3-bit numbers in fixed-width binary arithmetic, you'll get the "modular arithmetic" answer:

- 1 + 2 -> 3
- 4 + 5 -> 1

The calculations wrap around, because any answer larger than 7 cannot be represented with 3 bits. The wrapped-around answer is still meaningful:

• The answer we got is congruent (i.e., equivalent) to the real answer, modulo 8.

This is the modular arithmetic! The real answer was 9, but we got 1. And, both 9 and 1 leave remainder 1 when divided by 8.

consider 3-bit integers, which can represent integers from 0 to 7.

If you add or multiply two of these 3-bit numbers in fixed-width binary arithmetic, you'll get the "modular arithmetic" answer:

- 1 + 2 -> 3
- 4 + 5 -> 1

The calculations wrap around, because any answer larger than 7 cannot be represented with 3 bits. The wrapped-around answer is still meaningful:

• The answer we got is congruent (i.e., equivalent) to the real answer, modulo 8.

This is the modular arithmetic! The real answer was 9, but we got 1. And, both 9 and 1 leave remainder 1 when divided by 8.

• The answer we got represents the lowest 3 bits of the correct answer For 4+5, we got 001, while the correct answer is 1001.

consider 3-bit integers, which can represent integers from 0 to 7.

If you add or multiply two of these 3-bit numbers in fixed-width binary arithmetic, you'll get the "modular arithmetic" answer:

- 1 + 2 -> 3
- 4 + 5 -> 1

The calculations wrap around, because any answer larger than 7 cannot be represented with 3 bits. The wrapped-around answer is still meaningful:

• The answer we got is congruent (i.e., equivalent) to the real answer, modulo 8.

This is the modular arithmetic! The real answer was 9, but we got 1. And, both 9 and 1 leave remainder 1 when divided by 8.

• The answer we got represents the lowest 3 bits of the correct answer For 4+5, we got 001, while the correct answer is 1001.

Imagine infinite number line:

+ 001 010)+(011)+(100)+(101)+(110)+(111 000 1000)+

Imagine infinite number line:

000 001 010)→(011)→(100)→(101)+(110 111 1000)+ H

Then curl the number line into a circle so that 1000 overlaps the 000:



the adder for unsigned integers can be used for signed integers too, exactly as it is!

Binary	Unsigned value	Signed value
000	0	0
001	1	1
010	2	2
011	3	3
100	4	-4
101	5	-3
110	6	-2
111	7	-1

You can interpret those eight values as signed or unsigned.

It is called "two's complement" because to negate an integer, you subtract it from 2^{N} .

For example, to get the representation of -2 in 3-bit arithmetic, you can compute 8 - 2 = 6, and so -2 is represented in two's complement as 6 in binary: 110.

It is called "two's complement" because to negate an integer, you subtract it from 2^{N} .

For example, to get the representation of -2 in 3-bit arithmetic, you can compute 8 - 2 = 6, and so -2 is represented in two's complement as 6 in binary: 110.

another way to compute two's complement, which is easier to imagine implemented in hardware:

- 1. Start with a binary representation of the number you need to negate
- 2. Flip all bits
- 3. Add one

test.pas (/tmp) - VIM	×
var i, j: integer;	
begin	
i := -3; j := 5; M <mark>riteln</mark> (i * j)	
end.	
	A 7 7
"test.pas" 10L, 73C	ALL

1		test.s (/tmp) -	VIM		х
	Leaq	-16(%rsp),%rsp			
	movq	%rbx,-8(%rbp)			
	call	FPC_INITIALIZEUNITS			
	movw	<pre>\$-3,U_\$P\$PROGRAM_\$\$_I</pre>			
	movw	<pre>\$5,U_\$P\$PROGRAM_\$\$_J</pre>			
	call	fpc_get_output			
	movq	%rax,%rbx			
	movswq	U_\$P\$PROGRAM_\$\$_I,%rax			
_	movswq	U_\$P\$PROGRAM_\$\$_J,%rdx			
Ũ	imulq	%rax,%rdx			
	movq	%rbx,%rsi			
	movl	\$0,%edi			
	call	fpc_write_text_sint			
	call	FPC_IOCHECK			
	movq	%rbx,%rdi			
	call	fpc_writeln_end			
	call	FPC_IOCHECK			
	call	FPC_DO_EXIT			
	movq	-8(%rbp),%rbx			
	leave				
	ret				
•	Lc2:				
1	test.s	[asm,utf-8,unix] [F3]: PasteMode off	18,2-5 1	.0%
	set nolist				

	t.c (/tmp) - VIM	
<pre>#include <stdio.h></stdio.h></pre>		
int main() {		
<mark>u</mark> nsigned int a, b; a = -3; b = 5;		
printf ("%d\n", a * b);		
}		
~		
1 t.c [c,utf-8,unix]	[F3]: PasteMode off	6,3

×

All

		t.s	(/tmp) - VIM		×
	pushq	%rbp			
	.cfi de	f cfa offset 16			
	.cfi of	fset 6, -16			
	movq	%rsp, %rbp			
	.cfi_de	f_cfa_register 6			
	subq	\$16, %rsp			
	movl	\$-3, -4(%rbp)			
	movl	\$5, -8(%rbp)			
	movl	-4(%rbp), %eax			
	imull	- <mark>8</mark> (% <mark>rbp</mark>), % <mark>eax</mark>			
	movl	%eax, %esi			
	movl	\$.LC0, % edi			
	movl	\$0, %eax			
	call	printf			
	movl	\$0, %eax			
	leave				
	.cfi_de	f_cfa 7, 8			
	ret				
	.cfi_en	dproc			
	LFE0:				
	.size	main, main			
	.ident	"GCC: (Funtoo 6.3.0) 6.3	.0"		
1	t.s [as	m,utf-8,unix]	[F3]: PasteMode off	11,2-5	90%
	set nolist				
085.00					-

int a = int.Parse(Console.ReadLine()); int b = int.Parse(Console.ReadLine()); Console.WriteLine(a * b);

 0000004f
 call
 79084EA0

 00000054
 mov
 ecx,eax

 00000056
 imul
 esi,edi

 00000059
 mov
 edx,esi

 0000005b
 mov
 eax,dword ptr [ecx]

uint a = uint.Parse(Console.ReadLine()); uint b = uint.Parse(Console.ReadLine()); Console.WriteLine(a * b);
 0000004f
 call
 79084EA0

 00000054
 mov
 ecx,eax

 00000056
 imul
 esi,edi

 00000059
 mov
 edx,esi

 0000005b
 mov
 eax,dword ptr [ecx]

The IMUL instruction does not know whether its arguments are signed or unsigned, and it can still multiply them correctly!

int a = int.Parse(Console.ReadLine()); int b = int.Parse(Console.ReadLine()); Console.WriteLine(a * b);

 0000004f
 call
 79084EA0

 00000054
 mov
 ecx,eax

 00000056
 imul
 esi,edi

 00000059
 mov
 edx,esi

 0000005b
 mov
 eax,dword ptr [ecx]

uint a = uint.Parse(Console.ReadLine()); uint b = uint.Parse(Console.ReadLine()); Console.WriteLine(a * b);

0000004f	call	79084EA0
0000054	mov	ecx,eax
00000056	imul	esi,edi
00000059	mov	edx,esi
0000005b	mov	eax,dword ptr [ecx]

IMUL is for signed multiplications. MUL is for unsigned. IMUL checks for overflows and sets the overflow processor flag.

127	0111 1111	0111 1111	0111 1111
	0000 0001		
1	0000 0001	0000 0001	0000 0001
0	0000 0000	0000 0000	0000 0000
-0	1000 0000	1111 1111	
-1	1000 0001	1111 1110	1111 1111
-2	1000 0010	1111 1101	1111 1110
-3	1000 0011	1111 1100	1111 1101
-4	1000 0100	1111 1011	1111 1100
-5	1000 0101	1111 1010	1111 1011
-6	1000 0110	1111 1001	1111 1010
-7	1000 0111	1111 1000	1111 1001
-8	1000 1000	1111 0111	1111 1000
-9	1000 1001	1111 0110	1111 0111
-10	1000 1010	1111 0101	1111 0110
-11	1000 1011	1111 0100	1111 0101
-127	1111 1111	1000 0000	1000 0001
-128			1000 0000

13	0013
12	0012
11	0011
10	0010
9	0009
8	0008
2	0002
1	0001
0	0000
-1	9999
-2	9998
-3	9997
-4	9996
-9	9991
-10	9990
-11	9989
-12	9988

Pascal:

C:

int convert(int a) {
 if (a < 0)
 a = ~(-a) + 1;
 return a;
}</pre>

}

Pros:

- Same cpu instructions for addition, substruction, multiplication (check overflow flags)
- No -0

Cons:

- Unusual for humans representation
- if you try to negate the lowest representable value, you get an overflow

Sign extention

10	0000 1010	0000 0000 0000 1010
-15	1111 0001	1111 1111 1111 0001